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OPTIMAL CONTROL OF SYSTEMS WITH UNCERTAINTY.(U)

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OPTIMAL CONTROL OF SYSTEMS WITH UNCERTAINTY

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# ABSTRACT

The research supported by AFOSR Grant 76-2923 has led to some significant results which can be used in the design of optimal controllers when disturbances are present. Sufficient conditions which a minmax control must satisfy were developed for systems with disturbances in the state equation or in the measurement of the initial state. From these conditions, constructive techniques were developed which can be used to generate minmax controllers.

The second major area of the research was controllability problems. Criteria for determining the controllability properties of constrained systems were derived. The criteria involve finite dimensional optimization problems and are amenable to computer implementation.

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## Sec. I. INTRODUCTION

The research conducted under AFOSR-Grant 76-2923 was mainly concerned with two general problems. The first is the optimal control of systems with uncertainty and the second is constrained controllability problems. A description of these problems and the results obtained are given in Section II and Section III, respectively.

In the course of the research, other problems, which do not fall neatly into these two categories, also arose. Section III describes these related problems and our progress toward solving them.

## Sec. II. Optimal Control of Systems With Uncertainty

In the control of complex systems, uncertainties will usually occur in the mathematical description of the system. For example, the differential equations describing the system may not be known exactly or it may not be possible to make exact measurements of the state of the system. Air Force systems such as air-to-air missile encounters with an aircraft or missile guidance systems are examples of such problems. Ignoring uncertainties in the design of controllers for these systems may result in the actual system performing poorly and inaccurately. Proper methods for analyzing systems with uncertainty are needed. The research conducted under AFOSR-Grant 76-2923 has addressed the problem of the optimal control of systems with uncertainty.

Our approach to these problems is to assume that nature is perverse and may choose the uncertainty to maximize the performance index which the controller is trying to minimize. For each control, there is a guaranteed performance and the optimal control is the one which achieves the best guaranteed performance. This approach leads quite naturally to the concept of minmax control. A minmax control has the appealing property of producing the best possible guaranteed performance. Unlike the stochastic approach to uncertainty, the minmax approach does not require that the statistics of the uncertainty

be known. This is advantageous since the statistics of the uncertainty are often difficult to estimate. Also, a minmax control may be more easy to determine and implement than a stochastic control.

While the main concern of the investigation has been with dynamic minmax problems, it was felt that it would be worthwhile to also investigate static problems. The reasons for this are two-fold. First, static problems are easier to solve than dynamic ones; yet the characteristics of the solutions of both problems have much in common. A deeper understanding of the static case is useful in the analysis of dynamic problems. Secondly, the condition for dynamic minmax problems analogous to Pontryagin's principle involves a static minmax problem and the results for static minmax problems are used in the dynamic case.

Necessary conditions and sufficient conditions for static minmax problems have been developed and are presented in [1]. Later, a simpler derivation of the necessary condition was reported in [2]. The necessary conditions, in the form of a Lagrange multiplier rule, can be used to determine candidates for the solution. The sufficient conditions can be used to verify whether a candidate is indeed the solution. Since the sufficient conditions involve a strengthening of the necessary conditions, they are easy to apply once a candidate has been obtained.

In some problems, the performance of the system cannot be measured by a single criterion alone, but multiple criteria are needed. The minmax results of [1] were extended to problems with multiple criteria in [3]. It is also shown there that solution candidates for the multicriteria case can be obtained by solving a related problem with a scalar criterion. This simplifies the effort needed to obtain solutions to problems with multiple criteria.

With dynamic systems, the uncertainty or disturbance may enter the system

through the state equations or through the initial conditions. First consider problems with time-varying uncertainty in the state equations. These problems arise when it is not possible to obtain an exact model of the system. Often the analysis is carried out by neglecting the uncertainty. However, this may be too idealized for the analysis to be valid and the actual system may not perform well. Thus the analysis must take into account the fact that the model is not exact and the minmax solution concept is an attractive way to treat the uncertainty since it assures the best possible guaranteed performance. In practice, nature will probably not be so perverse as to choose the disturbance to maximize the performance index and the system will perform better than predicted. However, if a control not having the minmax property is used, the system may perform decidedly worse than expected. Thus a control having the minmax property should be used in the design of systems with uncertainty when there is no a priori knowledge of the value of the uncertainty.

We have developed a sufficient condition which the minmax control must satisfy [4]. This condition also leads to a method for constructing a minmax control. It has been shown that the minmax control can be obtained by solving a related optimal control problem without uncertainty. Thus the well-developed techniques from deterministic optimal control theory can be used to solve problems with uncertainty via this related problem. This is thought to be a significant step in the direction of obtaining methods which can be readily used to solve problems with time varying uncertainty in the state equation. More recently, a generalization of the condition appeared in [5].

In [6], problems with uncertain initial conditions are treated. In these problems, the exact value of the initial state is not known. Instead, only an inexact measurement is available. This is often the case in realistic situa-

tions where, for example, due to hardware limitations, position and velocity cannot be measured exactly. All that is available is the measured values of those quantities which equal the true values plus or minus some error. Our results were obtained by using a transformation which transforms the original problem with uncertainty in the initial state to one with the initial state known but with parameter uncertainty in the state equation. The latter problem is simpler to solve. It can be shown that the solution of the new problem is a solution to the original one with uncertain initial condition. Through this observation, we are able to present a constructive technique for finding the minmax control and also a sufficient condition which can be used to verify that a control has the minmax property.

As a by-product, our techniques can also be used to solve some problems where there is parameter uncertainty in the state equation (rather than time-varying uncertainty). Parameter uncertainty often occurs in the system model when there is a lack of experimental data so that the exact values of the parameters in the model are unknown.

The above research on optimal control problems with uncertainty was directed toward problems where the performance of the system could be measured by a single, scalar performance index. However, in many practical problems one has to deal with multiple (and possibly conflicting) objectives. These multicriteria problems do not, in general, have a best solution in the sense that there is a control function which simultaneously minimizes all the performance measures. While there is no universally accepted solution concept for such problems, one would agree that a good solution must not be dominated by any other admissible solution. This leads to the concept of Pareto optimality as an optimality criterion for problems with multiple performance measures.

Various techniques are available for determining Pareto optimal solutions when there is no uncertainty in the system, but these results do not apply when disturbances are present in the system's differential equations. We have extended the techniques we developed for problems with disturbances and a scalar performance index to problems with multiple performance indices [7]. These results can be used to obtain minmax solutions with the Pareto optimal property when disturbances are present.

In summary, we derived sufficient conditions which the minmax control must satisfy and, using these conditions, constructive techniques were developed which can be used to generate minmax solutions. These methods are now available for solving minmax problems and can be used to analyze problems where there is uncertainty in the model or in the measurement of the initial state. They will aid in the design of systems where exact models of the system are not available or where exact measurements of the state of the system cannot be obtained.

### Sec. III. Constrained Controllability

A fundamental problem associated with the design of control systems is that of controllability. The controllability problem is to determine if there is a control, satisfying specified magnitude constraints, which steers the system to a given target from a particular initial state. In some cases, we may want to reach the target from every possible initial state and this is called the global controllability problem. The class of systems we have analyzed are those described by ordinary differential equations:

$$\dot{x}(t) = A(t)x(t) + f(t, u(t)).$$

Here  $x(t) \in R^m$  is the state and  $u(t) \in R^m$  is the control. Magnitude constraints are imposed by requiring  $u(t) \in \Omega$ , where  $\Omega$  is a prespecified set in  $R^m$ . The target set  $X$  is a given subset of  $R^n$ .



Methods for checking the controllability properties of a system have been developed. Our criteria have the advantage of being finite dimensional, rather than infinite dimensional. This advantage is quite important from a computational viewpoint. In addition, we have devised some procedures for determining a steering control. Our results have been reported for problems where the target set  $X$  is the origin [8], where  $X$  is a general closed, convex set [9], where  $X$  is affine [10], i.e.,  $X = \{x: Lx = a\}$  and where  $X$  is any arbitrary set [11]. These papers treat the global problem as well as the problem of controllability from a particular initial state. We have also obtained local controllability results [12].

Recently, we extended our techniques to problems where disturbances are present [13]. In these problems, we want to guarantee that a system can be steered to the target even though unknown disturbances may be acting. Criteria for analyzing such problems have been developed.

Closely related to the controllability problem are the holding problem and avoidance problem. Our study has derived conditions for determining whether a system can be held in a specified set or steered to avoid a given set [14].

#### Sec. IV. Other Results

In the course of our research, we have encountered several problems which do not fit into the areas discussed in Sec. II and Sec. III. In this section, we describe such problems and the results of our investigation.

The derivation of sufficient conditions for minmax control (see Sec. II) is based on methods from optimal control theory. While carrying out this derivation, an interesting feature of the direct sufficient conditions for optimal control problems was discovered. Several papers have suggested that

nonautonomous optimal control problems (problems where time appears explicitly in the different equations or in the performance index) be treated by making a transformation which eliminates the explicit dependence on time. However, we discovered that if such a transformation is made, one may not be able to reach any conclusion about sufficiency whereas sufficiency can be established if the problem is treated in its original, nonautonomous form. The details of the results are reported in [15].

Another problem which was investigated is the application of differential game theory to the problem of two firms attempting to maximize profits through advertising [16]. Various solution concepts, including minmax, are examined and the resulting advertising strategies and payoffs compared.

A distributed parameter optimal control problem was also studied [17]. Specifically, a solution to the problem of controlling the temperature distribution in a semi-infinite rod was determined. The solution was obtained by showing that this distributed parameter problem can be reformulated as a calculus of variation problem and applying results from the available theory for calculus of variation problems.

Another distributed parameter problem was also considered. This is the problem of reducing the drag force experienced by a solid body moving through a viscous fluid by the injection or suction of fluid at the body surface [18]. This is a multicriteria problem since there are three quantities which we want to minimize, namely, the square of the drag force, the square of the shear stress and the energy being expended through the use of mass flux. For this multicriteria problem, we have obtained the Pareto optimal solutions.

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3. "Static Multicriteria Problems: Necessary Conditions and Sufficient Conditions," Proceedings IFAC Symposium on Large Scale Systems, Udine, Italy, June 16-20, 1976.
4. "A Sufficient Condition for Minmax Control of Systems with Uncertainty in the State Equations," IEEE Trans. on Auto. Control, Vol. AC-21, No. 4, 1976.
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8. "Null controllability of Linear Systems with Constrained Controls," (with B. R. Barmish), SIAM J. on Control and Optimization, Vol. 18, no. 4, July, 1980.
9. "New Results on Controllability of Systems of the Form  $\dot{x}(t) = A(t)x(t) + f(t, u(t))$ ," (with B. R. Barmish), IEEE Trans. Auto. Control, Vol. AC-25, No. 3, June, 1980.
10. "Controlling a Constrained Linear System to an Affine Target," (with B. R. Barmish), IEEE Trans. Auto. Control, (submitted).
11. "Controlling a System to a Non-Convex Target," (with W.-G. Hwang), Proceedings 19th Conference on Decision and Control, Albuquerque, New Mexico, December 1980 (to appear).
12. "A Necessary and Sufficient Condition for Local Constrained Controllability of a Linear System," (with B. R. Barmish), IEEE Trans. Auto Control, Vol. AC-25, No. 1, Feb., 1980.
13. "Controllability Criteria for Systems with Disturbances," (with B. Elenbogen and B. R. Barmish), Proceedings Optimization Days 1980, Ecoles des Haute Etude Commerciales, Montreal, Canada, May 1980.

14. "Avoidance Control and Holding Control," (with B. R. Barmish), Proceedings 1980 Joint Automatic Control Conference, San Francisco, California, August 1980.
15. "A Note on the Use of Direct Sufficient Condition in Optimal Control Problems," Journal of Optimization Theory and Application, Vol. 23, No. 3 November 1977.
16. "Profit Maximization Through Advertising: A Nonzero Sum Differential Game Approach," (with G. Leitmann), IEEE Trans. Auto. Control, Vol. AC-23, No. 4, August 1978.
17. "Optimal Control of the End-Temperature in a Semi-Infinite Rod," (with W. E. Olmstead), Zeitschrift für angewandte Mathematik and Physik, Vol. 28, pp. 697-706, 1977.
18. "Optimal Blowing," (with W. E. Olmstead) SIAM Journal on Applied Math, Vol. 35, No. 3, November 1978.

## APPENDIX A. OTHER TECHNICAL ACTIVITIES

### A. JOURNAL PUBLICATIONS

The results of the research have resulted in eighteen publications which have appeared in the scientific literature. They are listed on pages (8-9) as References 1-18.

### B. CONFERENCES ATTENDED AND PAPERS PRESENTED

1. 1976 Joint Automatic Control Conference, Lafayette, Indiana, "Minimax Control of Systems with Uncertainty in State in the State Equations."
2. 1976 IEEE Conference on Decision and Control, Clearwater, Florida, "Minimax Control of Systems with Uncertainty in the Initial State." Also, chairman and organizer of a session on minmax problems.
3. 1977 Allerton Conference, Monticello, Illinois, "Multicriteria Optimization with Uncertainty in the Dynamics."
4. 1977 IEEE Conference on Decision and Control, New Orleans, Louisiana, Chairman, Session on "Games."
5. 1977 Joint Automatic Control Conference, San Francisco, California.
6. 1978 Allerton Conference, Monticello, Illinois, "A Necessary and Sufficient Condition for Constrained Controllability."
7. 1978 IEEE Conference on Decision and Control, San Diego, California, "Constrained Controllability."
8. 1979 Joint Automatic Control Conference, Denver, Colorado, "A Result on Controlling a Constrained Linear System to a Linear Subspace."
9. Optimization Days 1979, Montreal, Canada, "Controlling a System to a Target-Part I - Linear Systems with Origin as Target" and "Part II - Nonlinear Systems with a General Target."
10. 1979 IEEE Conference on Decision and Control, Fort Lauderdale, Florida.
11. Optimization Days 1980, Montreal, Canada, "Controllability Criteria for Systems with Disturbances."
12. 1980 Joint Automatic Control Conference, "Avoidance Control and Holding Control."
13. 1980 Conference on Decision and Control, "Controlling a System to a Non-Convex Target."

C. OTHER INVITED LECTURES

"Minmax Problems," Michigan State University, East Lansing, Mich., November 1977.

"Controlling Systems with Uncertainty," University of Rochester, Rochester, NY, November, 1978.

"Constrained Controllability," Michigan State University, East Lansing, Mich., April, 1980.

D. STUDENTS SUPPORTED

Bruce. E. Elenbogen, Ph.D., September 1980.  
Thesis Title: Constrained Controllability

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